QUANTUM FEEDBACK FOR PROTECTION OF SCHRÖDINGER CAT STATES

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We review the use of quantum feedback for combatting the decoherence of Schrödinger-cat-like states in electromagnetic cavities, with special emphasys on our recent proposal of an automatic mechanism based on the injection of appropriately prepared "probe" and "feedback" Rydberg atoms. In the latter scheme, the information transmission from the probe to the feedback atom is directly mediated by a second auxiliary cavity. The detection efficiency for the probe atom is no longer a critical parameter, and the decoherence time of the linear superposition state can be significantly increased using presently available technology.

1 Introduction

One of the most fundamental issues in quantum theory is how the classical macroscopic world emerges from the quantum substrate. This question is also an important point in the interpretation of quantum mechanics and it is still the subject of an intense debate [1,2]. The most striking example of this problem is given by the possibility, opened by quantum mechanics, of having linear superpositions of macroscopically distinguishable states, the so-called "Schrödinger-cat" states. Such paradoxical states are very sensitive to *decoherence*, i.e., the rapid transformation of these linear superpositions into the corresponding classical statistical mixture, caused by the unavoidable entanglement of the system with uncontrolled degrees of freedom of the environment [1]. The decoherence time depends on the form of system-environment interaction [3] but, in most cases, it is inversely proportional to the squared "distance" between the two states of the superposition [4]. It is then clear, that for macroscopically distinguishable states, the decoherence process becomes thus practically instantaneous [1]. Decoherence is therefore experimentally accessible only in the *mesoscopic* domain. In this case, one is able to monitor the progressive emergence of classical properties from the quantum ones. In this context, an important achievement has been obtained by Monroe *et al.* [5], who have prepared a trapped $^9Be^+$ ion in a superposition of spatially separated coherent states and detected the quantum coherence between the two localized states. However, the decoherence of the superposition state has not been observed in this experiment. The progressive decoherence of a mesoscopic Schrödinger cat has been monitored for the first time in the experiment of Brune *et al.* [6], where the linear superposition of two coherent states of the electromagnetic field in a cavity with classically distinct phases has been generated and detected.

Recently, the field of quantum information theory [7] has undergone an impressive development, and the study of decoherence has then become important not only from a fundamental, but also from a more practical point of view. All the quantum information processing applications rely on the possibility of performing unitary transformations on a system of N quantum bits (qubits), whose decoherence has to be made as small as possible. As a consequence, decoherence control is now a rapidly expanding field of investigation. In this respect, quantum error correction codes [8] have been developed in which the entangled superposition state of N qubits is "encoded" in a larger number of qubits. Assuming that only a fraction of qubits decoheres, it is then possible to reconstruct the original state with a suitable decoding procedure. These codes always require the entanglement of a large number of qubits, and will become practical only if quantum networks of tens of qubits become available. Up to now, the polarization states of three photons have been entangled at most [9]. Entangled states of two Rydberg atoms [10] or of two trapped ions [11] at most can be generated. Therefore, in the present experimental situation, it is more realistic to study complementary and more "physical" ways to deal with decoherence, based on the explicit knowledge of the specific process causing decoherence, which could be applied with very few degrees of freedom. This is possible, in particular, in quantum optics, when information is encoded in the quantum states of an electromagnetic mode (see for example [12]). In this case decoherence is caused by photon leakage. It is then possible to develop experimental schemes able to face photon leakage and the associated decoherence.

We have already shown in some recent papers [13,14,15,16,17] that a possible way to control decoherence in optical cavities is given by appropriately designed feedback schemes. Refs. [13] show that a feedback scheme based on the continuous homodyne measurement of an optical cavity mode is able to increase the decoherence time of a superposition state. In Ref. [16,17] a feedback scheme based on continuous photodetection and the injection of appropriately prepared atoms has been considered. This scheme, in the limit of very good detection efficiency, is able to obtain a significant "protection" of a generic quantum state in a cavity. In [15,17] this photodetection-mediated scheme has been adapted to the microwave experiment of Ref. [6] in which photodetectors cannot be used. The cavity state can only be indirectly inferred from measurements performed on probe atoms which have interacted with the cavity mode. Under ideal conditions, this adaptation to the microwave cavity case leads to a significant increase of the lifetime of the Schrödinger cat generated in [6]. However, this scheme suffers from two important limitations, making it very inefficient when applied under the actual experimental situation. It first requires the preparation of samples containing *exactly* one Rydberg atom sent through the apparatus. Up to now, the experimental techniques allow only to prepare a sample containing a random atom number, with a Poisson statistics. Two-atom events are excluded only at the expense of a low average atom number, lengthening the feedback loop cycletime [10]. The original scheme requires also a near unity atomic detection efficiency, which is extremely difficult to achieve even with the foreseeable improvements of the experimental apparatus.

Here we propose a significant improvement of the microwave feedback scheme described in [15,17]. This new version, using a direct transmission of the quantum information from the probe to the feedback atom, does not require a large detection efficiency, removing one of the main difficulties of the previous design. It however also requires sub-poissonian atom statistics. We show briefly how such atomic packets could be in principle prepared with standard laser techniques. Finally, our scheme improves the efficiency of the feedback photon injection in the cavity by using an adiabatic rapid passage.

2 Detection-mediated feedback

In this section we briefly review the original "stroboscopic" feedback scheme for microwave cavities proposed in [15,17]. This proposal is based on a very simple idea: whenever the cavity looses a photon, a feedback loop supplies the cavity mode with another photon, through the injection of an appropriately prepared atom. However, since there are no good enough photodetectors for microwaves, one has to find an indirect way to check if the high-Q microwave cavity has lost a photon or not. In the experiment of Brune *et al.* [6], information on the cavity field state is obtained by detecting the state of a circular Rydberg atom which has dispersively interacted with the superconducting microwave cavity. This provides an "instantaneous" measurement of the cavity field and suggests that continuous photodetection can be replaced by a series of *repeated* measurements, performed by non-resonant atoms regularly crossing the high-Q cavity.

The experimental scheme of the stroboscopic feedback loop is a simple modification of the scheme employed in Ref. [6]. The relevant levels of the velocity-selected atoms are two adjacent circular Rydberg states with principal quantum numbers $n = 50$ and $n = 51$ (denoted by $|g\rangle$ and $|e\rangle$, respectively) and a very long lifetime ($\simeq 30$ ms). The high-Q superconducting cavity is sandwiched between two low-Q cavities R_1 and R_2, in which classical microwave fields resonant with the transition between $|e\rangle$ and $|g\rangle$ can be applied.

The high-Q cavity C is instead slightly off-resonance with respect to the $e \to g$ transition, with a detuning $\delta = \omega - \omega_{eg}$, where ω is the cavity mode frequency and $\omega_{eg} = (E_e - E_g)/\hbar$. The Hamiltonian of the atom-microwave cavity mode system is the Jaynes-Cummings Hamiltonian,

$$H_{JC} = E_e|e\rangle\langle e| + E_g|g\rangle\langle g| + \hbar\omega a^\dagger a + \hbar\Omega\left(|e\rangle\langle g|a + |g\rangle\langle e|a^\dagger\right) \ , \qquad (1)$$

where Ω is the vacuum Rabi coupling between the atomic dipole on the $e \to g$ transition and the cavity mode. In the off-resonant case and perturbative limit $\Omega \ll \delta$, the Hamiltonian (1) assumes the dispersive form [17,18,19]

$$H_{disp} = \hbar\frac{\Omega^2}{\delta}\left(|g\rangle\langle g|a^\dagger a - |e\rangle\langle e|a^\dagger a\right) \ . \qquad (2)$$

A linear superposition state of two coherent states with opposite phases is generated when the cavity mode is initially in a coherent state $|\alpha\rangle$ and the Rydberg atom, which is initially prepared in the excited level $|e\rangle$, is subjected to a $\pi/2$ pulse both in R_1 and in R_2. In fact, when the atom has left the cavity R_2, the joint state of the atom-cavity system becomes the entangled state [6,17,19]

$$|\psi_{atom+field}\rangle = \frac{1}{\sqrt{2}}\left(|e\rangle\left(|\alpha e^{i\phi}\rangle - |\alpha e^{-i\phi}\rangle\right) + |g\rangle\left(|\alpha e^{i\phi}\rangle + |\alpha e^{-i\phi}\rangle\right)\right) \ , \qquad (3)$$

where $\phi = \Omega^2 t_{int}/\delta$ and t_{int} is the interaction time in C. A Schrödinger-cat state is then conditionally generated in the microwave cavity as soon as one of the two circular atomic states is detected.

As it was shown in Ref. [17], the stroboscopic feedback scheme works only for Schrödinger cat states with a definite parity, i.e. even or odd cat states, and therefore we shall restrict to $\phi = \pi/2$ from now on. In fact, when the cavity field initial state is a generic density matrix ρ, the state of the probe atom-field system after the two $\pi/2$ pulses and the $\phi = \pi/2$ conditional phase-shift can be written as [17]

$$\rho_{atom+field} = |e\rangle\langle e| \otimes \rho_e + |g\rangle\langle g| \otimes \rho_g + |e\rangle\langle g| \otimes \rho_+ + |g\rangle\langle e| \otimes \rho_- \ , \qquad (4)$$

where

$$\rho_e = P_{odd}\rho P_{odd} \tag{5}$$

$$\rho_g = P_{even}\rho P_{even} \ , \tag{6}$$

are the projections of the cavity field state onto the subspace with an odd and even number of photons, respectively, and the operators $\rho_{\pm}$ (whose expression is not relevant here) are given in [17]. Eq. (4) shows that there is a perfect correlation between the atomic state and the cavity field parity, which is the first step in an optimal quantum non demolition measurement of the photon number [20]. It is possible to prove that this perfect correlation between the atomic state and a cavity mode property holds only in the case of an exact $\phi = \pi/2$-phase shift sandwiched by two classical $\pi/2$ pulses in cavities R_1 and R_2 [17]. Moreover, the entangled state of Eq. (4) allows to understand how it is possible to check if the microwave cavity C has lost a photon or not and therefore to trigger the feedback loop, using atomic state detection only. The detection of e or g determines the parity of the field and, provided that the probe atomic pulses are frequent enough, indicates whether a microwave photon has left C or not. In fact, let us consider for example the case in which an odd cat state is generated (first atom detected in e): a probe atom detected in state e means that the cavity field has remained in the odd subspace. The cavity has therefore lost an *even* number of photons. If the time interval τ_{pr} between the two atomic pulses is much smaller than the cavity decay time γ^{-1}, $\gamma\tau_{pr} \ll 1$, the probability of loosing two or more photons is negligible and this detection of the probe atom in e means that no photon has leaked out from the high-Q cavity C. On the contrary, when the probe atom is detected in g, the cavity mode state is projected into the even subspace. The cavity has then lost an *odd* number of photons. Again, in the limit of enough closely spaced sequence of probe atoms, $\gamma\tau_{pr} \ll 1$, the probability of loosing three or more photons is negligible. A detection in g means that one photon has exited the cavity. Therefore, for achieving a good protection of the initial odd cat state, the feedback loop has to supply the superconducting cavity with a photon whenever the probe atom is detected in g, while feedback must not act when the atom is detected in the e state.

In Ref. [17] it has been proposed to realize this feedback loop with a switch connecting the g state field-ionization detector with a second atomic injector, sending an atom in the excited state e into the high-Q cavity. The feedback atom is put in resonance with the cavity mode by another switch turning on an electric field in the cavity C when the atom enters it, so that the level e is Stark-shifted into resonance with the cavity mode.

As it is shown in Ref. [17], if the probe atomic pulses are sufficiently frequent, this stroboscopic feedback scheme becomes extremely efficient and one gets a good preservation of an initial Schrödinger-cat state. However, if we consider the adaptation of this scheme to the present experimental apparatus of Ref. [6], we see that it suffers from two main limitations, which significantly decrease its efficiency. First of all the scheme is limited by the non-unit efficiency of the atomic state detectors ($\eta_{det} \simeq 0.4$), since the feedback loop is triggered only when the g-detector clicks. Most importantly, the above scheme assumes one has perfect "atomic guns", i.e. the possibility of having probe and feedback atomic pulses with *exactly one atom*. This is not experimentally achieved up to now. The actual experiment [6] has been performed using atomic pulses with a probability of having exactly one atom $p_1 \simeq 0.2$, close to the mean atom number in the sample. This low mean atom number has been chosen to minimize two–atom events. In this experimental situation, the proposed stroboscopic feedback scheme would have an effective efficiency $\eta_{eff} = \eta_{det} p_1^2 \simeq 0.016$, too low to get an appreciable protection of the Schrödinger cat state. In the next section we show how this scheme may be improved and adapted to the experimental apparatus employed in Ref. [6].

3 The new automatic feedback scheme

From the above discussion, it is clear that the limitations due to the non-unit efficiency of the atomic detectors could be avoided if we eliminate the measurement step in the feedback loop and replace it with an "automatized" mechanism preparing the correct feedback atom whenever needed. This mechanism can be provided by an appropriate conditional quantum dynamics, and this, in turn, may be provided by a second high-Q microwave cavity C', similar to C, replacing the atomic detectors, crossed by the probe atom first and by the feedback atom soon later, as described in Fig. 1.

The cavity C' is resonant with the transition between an auxiliary circular state i (an immediately lower circular Rydberg state), and level g. The interaction times have to be set so that both the probe and the feedback atom experience a π pulse when they cross the empty cavity C' in state g (or when they enter in state i with one photon in C'). This interaction copies the state of the probe atom onto the feedback atom, and thus removes any need for a unit detection efficiency.

This fine tuning of the interaction times to achieve the π-spontaneous emission pulse condition can be obtained applying through the superconducting mirrors of C' appropriately shaped Stark-shift electric fields which put the atoms in resonance with the cavity mode in C' only for the desired time. In this way, since C' is initially in the vacuum state, when the probe atom crosses

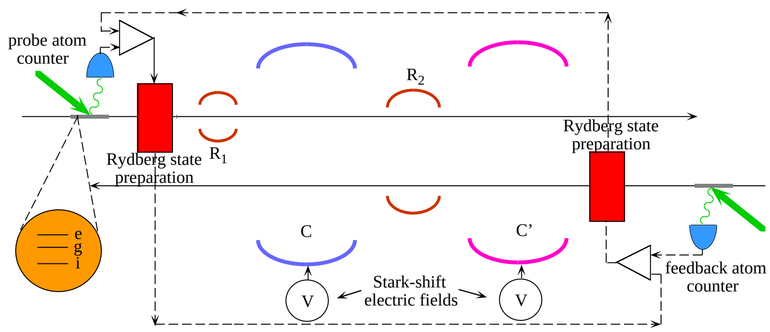

Figure 1: Schematic diagram of the autofeedback scheme proposed in this paper. R_1 and R_2 are the two cavities in which classical microwave pulses can be applied, C is the microwave cavity of interest and C' is the cavity automatically performing the needed correction. Electric fields can be applied at the superconducting mirrors of C and C' to Stark shift the Rydberg levels in order to tune the interaction times in C' and realize adiabatic transfer in C.

C' one has

$$|e\rangle_p|0\rangle_{C'} \rightarrow |e\rangle_p|0\rangle_{C'} \; , \tag{7}$$

$$|g\rangle_p|0\rangle_{C'} \rightarrow |i\rangle_p|1\rangle_{C'} \; . \tag{8}$$

Soon later a feedback atom enters C' in the state $|i\rangle_f$ and one has

$$|i\rangle_f|0\rangle_{C'} \rightarrow |i\rangle_f|0\rangle_{C'} \; , \tag{9}$$

$$|i\rangle_f|1\rangle_{C'} \rightarrow |g\rangle_f|0\rangle_{C'} \; . \tag{10}$$

In this way the cavity C' is always left disentangled in the vacuum state. The feedback atom exiting C' in $|g\rangle$ can be promoted to $|e\rangle$ before entering C, as required by the feedback scheme, by subjecting it to a π pulse in the classical cavity R_2 (see Fig. 1). Therefore, the conditional dynamics provided by C' eliminates any limitation associated to the measurement and leads to an "automatic feedback" scheme with unit efficiency in principle. Also the second important limitation of the stroboscopic feedback scheme of [15,17]—i.e.

that it requires exactly one probe and one feedback atom per loop—can be circumvented: A better control of the atom number, providing single atom events with a high probability, could be achieved by a modification of the Rydberg atoms preparation technique, in such a way that it is only triggered when the fluorescence detection signal (see Fig. 1) provides evidence of only one atom in the beam, implementing an atom counter.

Instead of preparing a random atom number at a given time, one thus prepares with a high probability a single Rydberg atom after a random delay. However, after a full quantum mechanical calculation and lengthy algebra, it is possible to determine the map of a generic feedback cycle, that is, the transformation connecting the states of the cavity field in C soon after the passage of two successive feedback atoms in C, which also takes into account the non-unit efficiency of the Rydberg state preparation. This map, which is reported elsewhere [21], allows us to study the dynamics of the Schrödinger-cat state in the presence of feedback, and to compare it with the corresponding dynamics in absence of feedback.

In Fig. 2 we show the Wigner function of the initial odd cat state (top) and its dynamics in presence (left) and in absence (right) of feedback. The comparison between the two performances is striking: in absence of feedback the Wigner function becomes quickly positive definite, while in the presence of feedback the quantum aspects of the state remain well visible for many decoherence times.

4 Conclusions

In this paper we have proposed a method to significantly increase the "lifetime" of a Schrödinger cat state of a microwave cavity mode. However, as it can be easily expected, most of the techniques presented here could be applied to the case of a generic quantum state of a cavity mode (see also Ref.[17]). After the first experimental evidences of decoherence mechanisms, decoherence control is an expanding field in quantum physics. An experimental realization of this realistic feedback scheme would be an important step in this direction.

Acknowledgments

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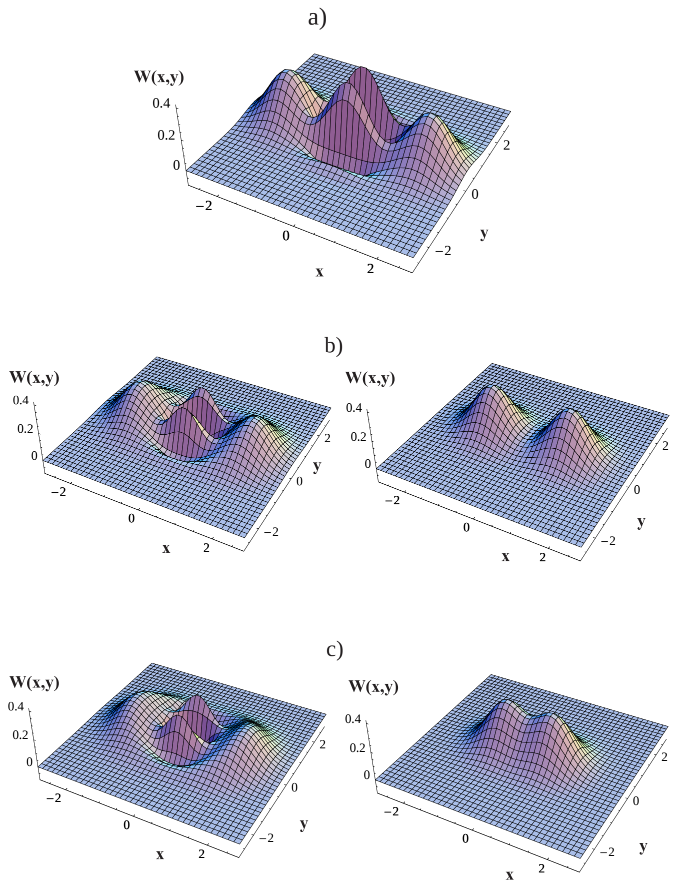

Figure 2: Wigner function of the initial odd cat state, $|\psi\rangle = N_-(|\alpha\rangle - |-\alpha\rangle)$, with $|\alpha|^2 = 3.3$ (a, top). Wigner function of the same cat state after 13 feedback cycles (b), corresponding to a mean elapsed time $\bar{t} \simeq 1/\gamma \simeq 6.6 t_{\rm dec}$, and after 25 feedback cycles (c) corresponding to a mean elapsed time $\bar{t} \simeq 2/\gamma \simeq 13 t_{\rm dec}$ (left). Wigner function of the same cat state after one relaxation time $t = 1/\gamma$ (b), and after two relaxation times $t = 2/\gamma$ (c), in absence of feedback (right).